Lifetime and decay of unstable particles in strong gravitational fields

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We consider here the decay of unstable particles in geodesic circular motion around compact objects. For the neutron, in particular, strong and weak decay are calculated by means of a semiclassical approach. Noticeable effects are expected to occur as one approaches the photonic circular orbit of realistic black-holes. We argue that, in such a limit, the quasi-thermal spectrum inherent to extremely relativistic observers in circular motion plays a role similar to the Unruh radiation for uniformly accelerated observers.

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I. INTRODUCTION

The possible decay of inertially stable particles due to strong gravitational fields has been considered recently. In particular, the proton decay, by weak and strong interactions, in uniformly accelerated trajectories[1, 2, 3] and in circular motion around compact objects[4], has been considered in great detail. The astrophysical implications of these results are now under investigation. The possible decay of accelerated protons, however, is not a new issue. It can be traced back to the works of Ginzburg and Zharkov[5] in the sixties, where processes of the type $p^+ \xrightarrow{a} n^0\pi^+$ were considered. At the same time, Zharkov[6] investigated the weak and strong decay of protons accelerated by an external electromagnetic field. (See [7] for a review.) Clearly, none of these processes would be allowed in the absence of external forces. We notice, however, that there are subtle differences between processes involving uncharged particles where the accelerations have gravitational and electromagnetic origins, see [4] for further details.

In this paper, we consider the decay and the lifetime of unstable particles in geodesic circular motion around spherically symmetrical compact objects. We evaluate, in particular, decay rates and lifetime for neutrons in relativistic circular motion according to the semiclassical approach introduced in [4]. Both the weak

$$n^0 \xrightarrow{a} p^+e^-\bar{\nu}_e \tag{1}$$

and the (inertially forbidden) strong

$$n^0 \xrightarrow{a} p^+\pi^- \tag{2}$$

channels are considered. Our results are compared to several ones obtained previously in the literature for the related processes $p^+ \xrightarrow{a} n^0e^+\nu_e$ and $p^+ \xrightarrow{a} n^0\pi^+$. As we will see, for realistic black-holes, noticeable effects are expected to occur for circular geodesics close to the photonic orbit $r = 3GM/rc^2$. Observers in these (unstable) circular orbits are necessarily in extremely relativistic motion ($v^2 \approx c^2$), and it is well known that they indeed realize the inertial vacuum as a quasi-thermal distribution of particles characterized by a temperature T in the range[8, 9]

$$\frac{\hbar a}{4\sqrt{3}c} \le kT \le \frac{\hbar a}{2\sqrt{3}c}, \tag{3}$$

where a stands for the effective Minkowskian centripetal acceleration for relativistic circular orbits. Our results suggest that the temperature (3) have in the present case the same central role played by Unruh temperature[10] in the analysis of uniformly accelerated particles as seen from Rindler observers[3]. (See [9] for a recent review.) Similar conclusions hold also for other unstable particles.

II. CIRCULAR GEODESICS AROUND COMPACT OBJECTS

The line element corresponding to a spherically symmetrical object of mass M is given by the Schwarzschild metric

$$ds^2 = -\left(1-2M/r\right)dt^2 + \left(1-2M/r\right)^{-1}dr^2 + r^2d\Sigma^2, \tag{4}$$

where $d\Sigma^2 = d\theta^2 + \sin\theta d\phi^2$. Natural unities are adopted hereafter. In this coordinate system[11], a particle of mass m in a circular timelike geodesic at radius r on the equatorial plane have energy per mass ratio given by

$$\mathcal{E}/m = (1-2M/r)\dot{t} = (1-2M/r)/\sqrt{1-3M/r}, \tag{5}$$

with the dot standing for s-derivative. Its angular momentum $L = r^2\dot{\phi}$ can be calculated directly from the

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definition of a timelike circular geodesic parameterized by the proper time s, leading finally to the following expression for the worldline of a timelike circular equatorial geodesic in Schwarzschild coordinates

$$x^a(s) = \left(\frac{s}{\sqrt{1-3M/r}}, r, \pi/2, s\sqrt{\frac{M/r^3}{1-3M/r}} \right). \quad (6)$$

Clearly, since the trajectory (6) is a geodesic, its acceleration $a^b = \dot{x}^c \nabla_c \dot{x}^b$ calculated with respect to the metric (4) vanishes identically. However, we will proceed here in a different manner. In the next section, we will consider quantum effects as realized by observers with circular trajectories as (6) in the Minkowski spacetime. An observer following the worldline (6) in Minkowski spacetime experiences a centripetal acceleration

$$a = \sqrt{a_b a^b} = \frac{M/r^2}{1-3M/r}. \quad (7)$$

On the other hand, in Minkowski spacetime the worldline of a particle in a uniform circular motion on the equatorial plane with angular velocity Ω is given by

$$x^a(\tilde{s}) = (t, r, \pi/2, \Omega t)\,, \quad (8)$$

from which one has immediately $\dot{x}^a = \gamma(1,0,0,\Omega)$ and $a = \sqrt{a_b a^b} = r\gamma^2\Omega^2$, where the constant $\gamma = dt/d\tilde{s} = (1-r^2\Omega^2)^{-1/2}$ corresponds to the Lorentz factor. The angular velocity Ω is to be determined by imposing the centripetal acceleration (7) for the trajectory (8), yielding

$$\Omega = \sqrt{\frac{M/r^3}{1-2M/r}} \quad (9)$$

and the following Lorentz factor

$$\gamma = \sqrt{\frac{1-2M/r}{1-3M/r}}. \quad (10)$$

The treatment of the quantum effects realized by observers in the circular geodesic (6) of Schwarzchild spacetime by means of an effective Minkowskian circular trajectory is, of course, only an approximation. It is shown in [12], nevertheless, that the results obtained in a semiclassical approach assuming a Schwarzschild spacetime and a flat spacetime with external "Newtonian" attraction forces such that (7) and (9) hold differ by no more than 30%, if we restrict ourselves to the circular orbits with $r > 3M$. Since Schwarzschild spacetime is asymptotically flat, it is indeed natural that the emitted powers calculated in Minkowski and Schwarzschild spacetime agree when one considers circular motions with large r. In fact, as it is shown in [4] and [12], the power emitted by a particle in circular motion with radius r in Minkowski spacetime with angular velocity (9) is very close to that one emitted by a particle in a circular geodesic with the same radius r in a Schwarzschild spacetime, provided that $r > 6M$. This is in agreement with the well known fact that processes involving wavelengths with the same order of magnitude of the Schwarzschild radius need necessarily to be analyzed using fully curved spacetime calculations. Moreover, the acceleration defined in (7) has the additional desirable feature of being divergent at $r = 3M$, in accordance with previous works on geodesic emission[13], which established that near the photonic orbit the emitted power diverges. The angular velocity (9) mimics the main qualitative properties of the real Schwarzschild circular geodesics, justifying our assumption of circular trajectories in a flat spacetime with centripetal acceleration (7).

III. EMISSION RATES AND LIFETIMES

The semiclassical current formalism employed in [4] to the proton decay case consists basically in considering the proton and the neutron as distinct energy eigenstates $|p\rangle$ and $|n\rangle$ of a two-level system such that $\hat{H}_0|p\rangle = m_p|p\rangle$ and $\hat{H}_0|n\rangle = m_n|n\rangle$, where $\hat{H}_0$ is the proper Hamiltonian of the system, m_p and m_n are, respectively, the proton and neutron masses. The weak channel (1) is implemented by considering a vector current associated to the two-level system coupled to a quantized fermionic field (corresponding to the electron e^- and to the antineutrino $\bar{\nu}_e$) by means of the effective coupling constant $G_{\rm w}$, which is about the order of the Fermi coupling constant $G_F \approx 1.166 \times 10^{-5}\,{\rm GeV}^{-2}$, whereas the strong channel (2) involves a scalar current coupled to a quantized bosonic field (the pion π^-) by means of the effective coupling constant $G_{\rm s}$, of the order of the pion-nucleon-nucleon strong coupling $g^2_{\pi NN}/4\pi \approx 14$[14]. The currents are then specialized to the case of uniform circular trajectories with radius r and angular velocity Ω (and centripetal acceleration $a = r\gamma^2\Omega^2$) in Minkowski spacetime.

The proper decay rates corresponding to the weak (1) and to the strong (2) channels are given, respectively, by

$$\Gamma^{\rm w}_{n\to p} = -\frac{G^2 a^5}{8\pi^4} \oint d\lambda\, e^{i\delta} \frac{A_{(bc)} Z^b Z^c}{(Z_a Z^a)^2} \times \left(\frac{16}{\gamma^4 (Z_a Z^a)^2} + 4\frac{\mu^2}{\gamma^2 Z_a Z^a} \right) \quad (11)$$

and

$$\Gamma^{\rm s}_{n\to p} = -\frac{G^2 a}{4\pi^2} \oint d\lambda \frac{e^{i\delta}}{\gamma^2 Z_a Z^a}, \quad (12)$$

where $\mu^2 = (m_e^2 + m_\nu^2)/a^2$, m_e and m_ν being, respectively, the electron and neutrino masses, and $\delta = (m_n - m_p)/a$. We assume here $m_\nu = 0$. The rates (11) and (12) are obtained, respectively, from equations (3.31) and (3.16) of [4], where all the relevant details can be found. In particular, we have $Z^a = (-\lambda + i\epsilon, 0, -(2ra/\gamma)\sin(\Omega\lambda\gamma/2a), 0)$, where $0 < \epsilon \ll 1$ is a regulator. For the relativistic case ($\gamma \gg 1$), the terms involved in the integrations (11) and

(12) are[4]

$$A_{(bc)}Z^bZ^c \approx \frac{\lambda^2}{\gamma^4}\left(\frac{\lambda^4}{72}+\frac{\lambda^2}{12}+1\right), \tag{13}$$

$$Z_aZ^a \approx \frac{1}{12\gamma^2}\left(\lambda+i\sqrt{3}A_+\right)\left(\lambda+i\sqrt{3}A_-\right)\times \left(\lambda-i\sqrt{3}B_+\right)\left(\lambda-i\sqrt{3}B_+\right), \tag{14}$$

where $A_\pm = 1\pm\sqrt{1+2\epsilon/\sqrt{3}}$, $B_\pm = 1\pm\sqrt{1-2\epsilon/\sqrt{3}}$, see Fig. 1.

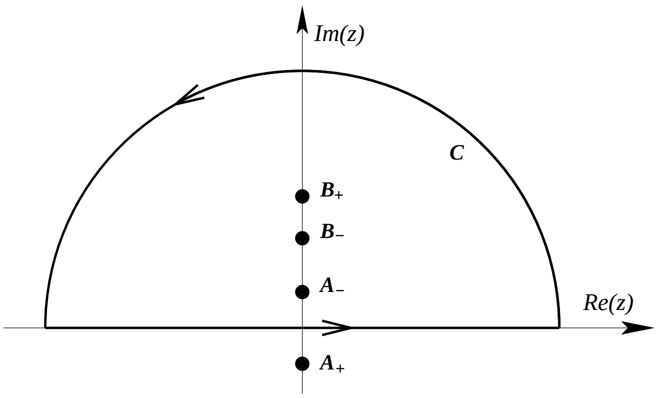

FIG. 1: Path used in the complex integrations (11) and (12). Both terms (17) and (18) come from the pole B_+. They coincide with the values calculated in [4] by integrating around the pole A_+. Notice that, for the processes considered there, the term δ has a different sign. The second terms in (15) and (16) come from the (degenerated) poles A_- and B_-.

In contrast to the case considered in [4], since $m_n > m_p$, one needs here to perform the complex integrations (11) and (12) along the path depicted in Fig. 1. We get, after taking the limit $\epsilon \to 0$,

$$\Gamma^{\rm w}_{n\to p} = \Gamma^{\rm w}_{p\to n} + \frac{G_{\rm w}^2 a^5\delta}{90\pi^3}\left(20+15\delta^2+3\delta^4-15\mu^2\left(1+\delta^2\right)\right) \tag{15}$$

and

$$\Gamma^{\rm s}_{n\to p} = \Gamma^{\rm s}_{p\to n} + \frac{G_{\rm s}^2 a\delta}{2\pi}, \tag{16}$$

where $\Gamma^{\rm w}_{p\to n}$ and $\Gamma^{\rm s}_{p\to n}$ correspond, respectively, to the proper decay rates associated to the inverse processes considered in [4],

$$\Gamma^{\rm w}_{p\to n} = \frac{G_{\rm w}^2 a^5 e^{-2\sqrt{3}\delta}}{1728\pi^3}\left[49\sqrt{3}+102\delta+30\sqrt{3}\delta^2+12\delta^3 -\mu^2\left(39\sqrt{3}+90\delta+36\sqrt{3}\delta^2\right)\right] \tag{17}$$

and

$$\Gamma^{\rm s}_{p\to n} = G_{\rm s}^2 a e^{-2\sqrt{3}\delta}/(8\sqrt{3}\pi). \tag{18}$$

The approximations involved in the derivation of the expressions (11) and (12) require, respectively, that the centripetal acceleration a obeys $m_e < a < m_p$ and $m_\pi < a < m_p$, where m_π stands for the pion π^- mass. We notice that, for accelerations $a \gg m_p$, the no-recoil hypothesis is violated[4] and, hence, our approximation breaks down. The neutron proper lifetime associated with the decay rates (15) and (16) are given simply by $\tau_n^{\rm w} = 1/\Gamma^{\rm w}_{n\to p}$ and $\tau_n^{\rm s} = 1/\Gamma^{\rm s}_{n\to p}$. Our results assume a particularly simple form if one considers the proton and neutron lifetime ratio, namely

$$\tau_p^{\rm s}/\tau_n^{\rm s} = 1+12\delta e^{2\sqrt{3}\delta}, \tag{19}$$

and

$$\tau_p^{\rm w}/\tau_n^{\rm w} = 1+\frac{96}{5}\delta e^{2\sqrt{3}\delta}\frac{P(\delta,\mu)}{Q(\delta,\mu)}, \tag{20}$$

where $P(\delta,\mu)$ and $Q(\delta,\mu)$ are polynomials easily obtained from (15) and (17). It is clear that for large a, both ratios obey

$$\tau_p/\tau_n \approx 1+O(a^{-1}). \tag{21}$$

The strong channel is expected to be the dominant decay mode for neutrons with centripetal accelerations a such that $m_\pi < a < m_p$. Taking into account that $m_\pi \approx$ 139.57 MeV, $m_n \approx 939.56$ MeV and that $m_p \approx 938.27$ MeV, we have that the proton and the neutron lifetimes differ by no more than 1% to 10% in the range of accelerations where the strong channel dominates. The weak channel, on the other hand, dominates for smaller accelerations $m_e < a < m_\pi$. (We remind that $m_e \approx 0.51$ MeV.) From (19) and (20), it is clear that for

$$a \gg a_c = m_n - m_p \approx 1.29\,\text{Mev} \tag{22}$$

the asymptotic expression (21) holds accurately. The meaning of the "critical" acceleration a_c will be discussed in the last section. Here, we mention only that a_c belongs to the range where the weak channel dominates. In fact, for $m_e < a < a_c$, a significative difference between the proton and the neutron lifetime is observed.

A proper acceleration of the order of 1 MeV is extremely high. For sake of comparison, protons in the CERN Large Hadron Collider have proper acceleration $a \approx 10^{-8}$ MeV[4]. In order to compare a_c with centripetal accelerations induced by realistic black-holes, we cast (7) in the form

$$a \approx 1.34\times 10^{-16}\left(\frac{M_\odot}{M}\right)\frac{\left(GM/rc^2\right)^2}{1-3GM/rc^2}, \tag{23}$$

where a is now measured in MeV and the numerical constant corresponds to $\hbar c^3/GM_\odot$. Hence, realistic black holes ($M \approx M_\odot$) will induce centripetal accelerations of the MeV order only for those circular orbits very close to the photonic orbit $r = 3GM/c^2$. Smaller black holes, nevertheless, can induce considerably higher centripetal accelerations for the (unstable) circular orbits located between the photonic orbit and the last stable circular orbit

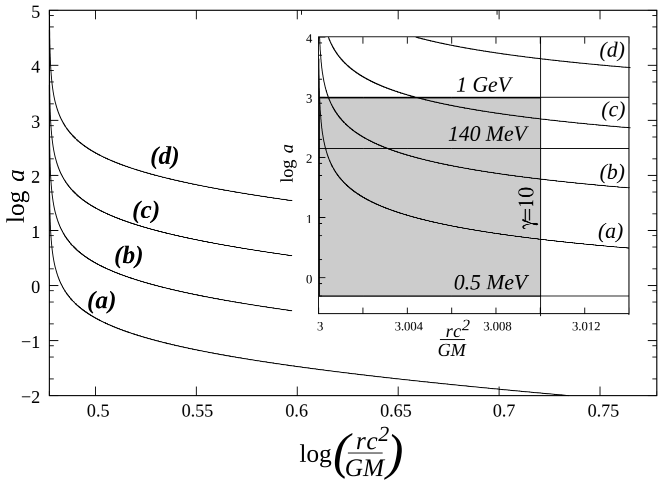

FIG. 2: Centripetal acceleration, in MeV, given by equation (23), for circular trajectories such that $3GM/c^2 < r \leq 6GM/c^2$. The curves (a) to (d) correspond, respectively, to the ratios $M/M_\odot = 10^{-15}, 10^{-16}, 10^{-17}$, and 10^{-18}. We recall that, for the neutron decay, the approximations involved in the derivation of our results require $a < m_p \approx 938.27$ MeV (no-recoil hypothesis) and $a > m_e \approx 0.51$ MeV (weak channel) or $a > m_\pi \approx 139.57$ MeV (strong channel). The hypothesis of an extremely relativistic motion ($\gamma \gg 1$), on the other hand, requires orbits close to the photonic one, see equation (24). In the detail, the hatched area denotes the region of validity of our approximations, assuming $\gamma > 10$.

at $r = 6GM/c^2$, see Figure 2. On the other hand, the hypothesis of an extremely relativistic motion ($\gamma \gg 1$) adopted in the approximations (13) and (14) requires circular trajectories close to the photonic orbit, since from (10) one has

$$\frac{rc^2}{GM} = \frac{3\gamma^2 - 2}{\gamma^2 - 1}. \tag{24}$$

Figure 3 depicts the lifetime for a neutron due to weak decay in circular orbits with $3GM/c^2 < r \leq 6GM/c^2$ around small black holes. As expected, the smaller is the black hole, the larger is the reduction in the particle lifetime. The semiclassical approach used here can be applied for other unstable particles as well. Particularly interesting is the muon weak decay $\mu^- \xrightarrow{a} e^- \bar{\nu}_e \nu_\mu$, which can also be described by a vector current based on a two-level system coupled to quantized fermions (the neutrinos $\bar{\nu}_e$ and ν_μ) by means of a coupling constant of the same order than G_F. Since neutrinos have very small masses, the rate (15) for the muon weak decay is accurate for accelerations in the range $0 < a < m_e \approx 0.51$ MeV. (We remind that $m_\mu \approx 105.7$ MeV.) Figure 4 depicts the lifetime of a muon in geodesic circular orbits of small black holes such that $3GM/c^2 < r \leq 6GM/c^2$

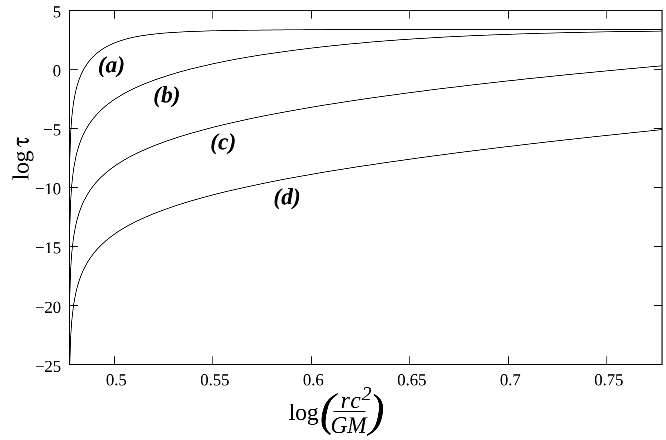

FIG. 3: Neutron proper lifetime τ, in seconds, for circular trajectories such that $3GM/c^2 < r \leq 6GM/c^2$. The curves (a) to (d) correspond, respectively, to the ratios $M/M_\odot = 10^{-16}, 10^{-17}, 10^{-18}$, and 10^{-19}. The (free) neutron inertial lifetime is approximately 886 s. The validity region of our approximations is depicted in Fig. 2.

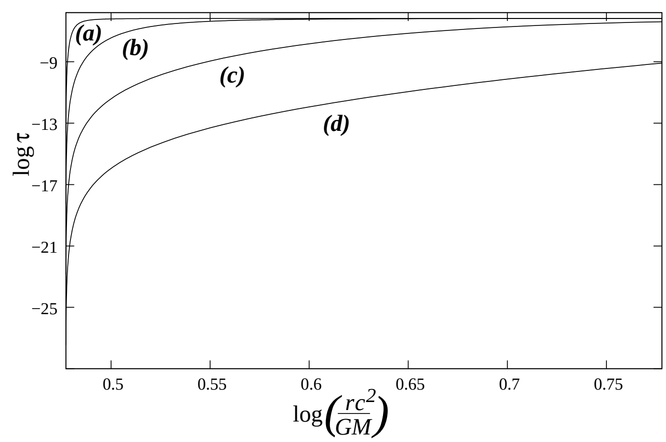

FIG. 4: Muon proper lifetime τ, in seconds, for circular trajectories such that $3GM/c^2 < r \leq 6GM/c^2$. The curves (a) to (d) correspond, respectively, to the ratios $M/M_\odot = 10^{-17}, 10^{-18}, 10^{-19}$, and 10^{-20}. The muon inertial lifetime is about 2.2×10^{-6}s and the branching ratio corresponding to the inertial process $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is greater than 98%. The validity region of our approximations is depicted in Fig. 2.

IV. DISCUSSION

As Figures 3 and 4 show, small black holes are necessary in order to induce sensitive alterations in the proper lifetime of unstable particles in circular orbits. However, noticeable effects do occur in the vicinity of the photonic orbit for realistic black hole. Despite that the analysis presented here is restricted to the (unstable) circular geodesics close to $r = 3M$, it can give some hints about the behavior of particles in more realistic situations. As equations (19) and (20) reveal, free protons and neutrons in geodesic circular motion close to the

photonic orbits have comparable proper lifetime. This situation is completely different from the inertial one, and its implication to particle physics in the vicinity of black holes has not been sufficiently studied yet. A similar conclusion holds for the muon. For a accelerations $a \gg a_c = 2\sqrt{3}(m_\mu - m_e) = 364.4$ MeV, neglecting possible effects of back-reaction[4], the muon and the electron, which in such a case can indeed decay by the inverse process $e^- \xrightarrow{a} \mu^- \bar{\nu}_\mu \nu_e$, have comparable lifetimes.

In order to grasp the meaning of the temperature (3), let us consider the case of the weak decay of protons and neutron in uniformly accelerated motion, where the Unruh temperature[10] $T_U = a/2\pi$ is known to play a central role[3, 9]. The proton and neutron lifetime ratio for this case can be obtained from the decay rates (3.13) and (3.17) of reference [2],

$$\tau_p^{\mathrm{w}}/\tau_n^{\mathrm{w}} = e^{2\pi\delta}, \tag{25}$$

which also has the asymptotic form (21) for large values of a. Notice that, in this case, we have exactly the same critical acceleration a_c of (22).

In the case of the uniformly accelerated motion, one can describe the decay of protons and neutrons as seen by comoving Rindler observers. The key point here is that Rindler observers realize the inertial vacuum as a thermal state with temperature $T_U = a/2\pi$. Heuristically, one can imagine the two-level system in thermal equilibrium with the Unruh radiation associated with the quantized fields in question. For our two-level system in thermal equilibrium at temperature T, the probability of occupation of the proton $|p\rangle$ and neutron $|n\rangle$ states are, respectively

$$N_p = \frac{e^{-m_p/T}}{e^{-m_p/T} + e^{-m_n/T}}, \quad N_n = \frac{e^{-m_n/T}}{e^{-m_p/T} + e^{-m_n/T}}. \tag{26}$$

The ratio $N_p/N_n = e^{(m_n - m_p)/T}$ diverges for $T \to 0$, indicating that for low temperatures, the system is likely to be in its fundamental state. However, for temperatures $T \gg (m_n - m_p)$ the ratio tends to 1, indicating that the system can be in the states $|p\rangle$ or $|n\rangle$ with equal probability. In other words, the transitions $|p\rangle \to |n\rangle$ and $|n\rangle \to |p\rangle$ become equally probable for high temperatures, shedding some light in the expression (25). For linear accelerations a such that the associate Unruh temperature $T_U = a/2\pi$ is much higher than the energy gap $m_n - m_p$, it is natural to expect that protons and neutrons have the same lifetime, since both transitions of the two level systems are equally probable. The lifetime ratios (19) and (20) suggest something similar to the case of uniform circular trajectories. They can be understood if one considers the two-level system in equilibrium with the quasi-thermal radiation with temperature (3) associated with the quantized fields in question, confirming the view that observers in relativistic circular motion with centripetal acceleration a do realize the inertial vacuum as a quasi-thermal state with temperature (3) for the extremely relativistic case[8, 9]. We stress that this state is quasi-thermal in the sense that it can be described by a temperature that varies slowly with the energy gap ΔE of the two-level system, monotonically from $(\pi/2\sqrt{3})T_U$ to $(\pi/\sqrt{3})T_U$, corresponding respectively to low and to high values of $\Delta E/a$ [9, 15, 16]. For the neutron decay, for instance, $\Delta E = m_n - m_p \approx 1.29$ MeV, implying that for circular trajectories close to the photonic orbit $r = 3GM/c^2$, the quasi-thermal state is indeed characterized by a temperature close to the lower bound of (3). For the muon decay, on the other hand, the temperature is closer to the upper bound of (3). A definitive answer to this problem, however, must necessarily face the subtle issue of quantum thermal distributions for rotating systems[17].

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